VARIATION AND CORRELATION IN RAYS AND DISK FLORETS OF ASTER FASTIGIATUS[1]

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(WITH FOUR FIGURES)

Through various researches on the variation of Compositae, Umbelliferae, etc., LUDWIG (**1, 2, 3, 5, 6, 7, 10**) came to the conclusion that multimodal curves prevail in plants, and that the mode always falls on some member of the Fibonacci series, or often on some member of SCHIMPER-BRAUN'S accessory series, that is, on the so-called "Nebenzahlen." When a mode comes on a number other than that in the principal and accessory series, he considers this as a *Scheingipfel*, in which two curves with modes on two adjacent classes are combined.

On the other hand, in the variation of the rays of *Chrysanthemum Leucanthemum*, LUCAS (**8**) obtained the principal mode on 22, a result which is an exception to LUDWIG'S rule, but which seemed to find confirmation later in the researches of SHULL (**12**) and TOWER (**13**). The curves of these two authors, however, were altogether multimodal and obtained from comparatively scanty material. For example, SHULL doubled the classes on account of the sparseness of his material, so that the apparent multimodal curves changed to monomodal ones. It may not be useless, therefore, to examine the applicability of LUDWIG'S rule in the variation of the rays and disk florets of *Aster fastigiatus* Fisch. and Mey., using a tolerably large number of variates.

The true significance of two different results in LUCAS'S calculation was not clearly known until TOWER'S discussion (**13**) appeared. According to him, the number of rays of *Chrysanthemum Leucanthemum* decreases continuously during the flowering season (27.87–21.38 between July 5 and July 30), and LUCAS'S curve and his own were not influenced by the difference of the place-modes.

A little before the publication of TOWER'S work, SHULL'S admirable paper (**12**) on the variation and correlation in the bracts, rays, and

[1] Preliminary note.

disk florets of *Aster Shortii*, *A. novae-angliae*, *A. puniceus*, and *A. prenanthoides* appeared. In this he recognized clearly the decrease of the number of the bracts, rays, and disk florets of *Aster prenanthoides* in the successive collections.[2]

The cause of the seasonal change of flowers in the multicipital Compositae is probably partly due to a change in nutrition, as HAACKE (4) and SHULL (12) have suggested; but an environmental condition seems to influence this decrease also. Recently Dr. KORIBA (16) of this laboratory found favorable material in the single-headed *Arnica unalaschensis*, which he collected on Mt. Hakkôda (Aomori), because the mean number of the rays changed exclusively from environmental changes. According to him, the mean increased from 14.389 to 15.741 between July 29 and August 5, for this was the season of thawing snow, and hence a favorable condition; then it decreased continuously to 15.081 on August 30.

During the summer of 1909 my attention was called to large patches of *Aster fastigiatus* on the banks of the Tone River, near Kokoku, Simosa Province, about thirty miles east of Tokyo. The number of rays and of disk florets was rather small, so that I was induced to calculate the coefficient of correlation between them. The present preliminary note seeks to verify LUDWIG'S rule in the variation of these rays and disk florets, and to find the seasonal change in rays and the range of correlation between rays and disk florets.

The material was taken from three patches scattered through a grassy field (about 100 meters square) along the river bank. The environmental condition of these three patches was almost the same, and seems not to affect in any degree the variation of the flowers; therefore, only the difference of time seems to account for the differences in the three collections. The first collection was made in the eastern part of the field, the second in the western, and the third in the middle. In collecting specimens care must be taken, for random collection seems to give a little larger value than picking all flowers from every individual (LUCAS 14); and the latter method, adopted in this investigation, may probably avoid personal error. All flowers that were injured by insects or that were extremely old or young were rejected.

[2] From September 27 to October 8 the mean number of rays decreased from 30.769 to 26.335.

As shown later, the class range in an individual is very restricted, and falls perhaps on one part of the curve of the racial variation. For this reason a more reliable result can be obtained by taking into account the flowers from an equal number of individuals, rather than an equal number of flowers from an unequal number of individuals. Hence I have counted in all cases all the flowers in ninety plants. The calculation has been made wholly according to DAVENPORT (**15**), and the results obtained are as follows:

Variations of rays

The first collection was made August 7, 1909, when the flowers had first begun to bloom, and the following data were obtained:

Number	Range	Mode	A	σ	C
1392	10–31	17	17.921 $\pm$0.051	2.831 $\pm$0.032	15.799

The curve (*fig. 1*) of this variation gives the positive skewness (PEARSON **9**, p. 408) 0.325, the mean value being very near 18. On the mode 17 fall 17.10 per cent and on 18 fall 16.38 per cent of all variates.

The second collection was made August 11, with the following results:

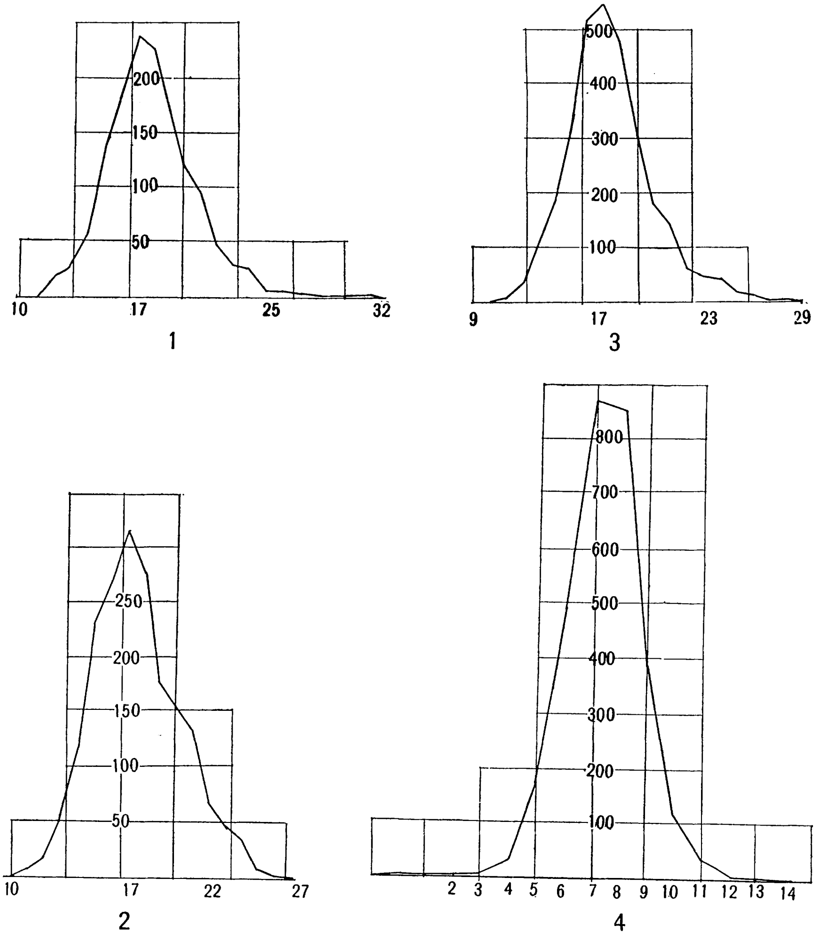
Number	Range	Mode	A	σ	C
1904	10–26	17	17.606 $\pm$0.041	2.618 $\pm$0.029	14.872

The percentage of variates occurring on mode 17 became a little less (16.55 per cent). This is due to the fact that the ordinates of the two classes 18 and 16 come nearly to the same height. The positive skewness became 0.231; the curve lessened the degree of asymmetry on account of the diminution in number of right-hand abscissae (*fig. 2*).

The third collection was made August 19, with the following results:

Number	Range	Mode	A	σ	C
2959	10–28	17	17.339 $\pm$0.031	2.521 $\pm$0.022	14.542

In this collection the range became somewhat wider than in the second, but all other elements decreased continuously. The mode came near to class 16, and the point of inflection of the curve came



FIGS. 1–4

nearer to the origin than in the two foregoing collections. The variates falling on the mode 17 increased to 18.28 per cent, and the positive skewness became only 0.134 (*fig. 3*).

On comparing the above three collections, it can be seen easily that the constants of variation decrease continuously from the begin-

ning to the end of the flowering season. The mean value of the rays decreased from 17.921 to 17.339 during twelve days, but no change of the mode was observed, the mode always holding its position on 17, which does not belong to the Fibonacci series. However, the oscillation of the mode between 18 and 16 may be observed from the first and third collections.

Variation of disk florets

This material was taken from the same flowers used in the third collection, and the constants calculated are as follows:

Number	Range	Mode	A	σ	C
2959	0–13	7–8	7.401 $\pm$0.017	1.329 $\pm$0.012	17.963

LUDWIG'S rule did not hold in this case, since the mode falls on 7 rather than on 8, although 7 is a member of SCHIMPER-BRAUN'S accessory series.

Individual correlation

I call briefly the correlation between the organs of the same individual the "individual correlation." PEARSON says (**9**, p. 392): "We have seen that the racial variation is greater than the individual variation, that capsules on the same poppy plant are more alike to each other than they are to the capsules of a second plant, or the leaves of one beech tree to each other than to those of a second beech tree. The resemblance of the like organs of the same individual is a special case of correlation, and we now want a quantitative measure of such correlation." I have tried, therefore, to find the value of the individual correlation, if such exists in my collections, and to throw some light upon this problem. For this purpose, I calculated the number (*R*) of classes and the difference (*D*) of the minimum and maximum classes in each individual variation, using the three collections mentioned above.

In the racial variation the class range was 10–31, that is, there occurred 21 different classes (one deficient in 11); while in the individual variation, as the above result (*R*) shows, only 2–10 different classes (rarely more than 6) occur, and curiously enough these classes happen to be almost continuous, as *D* clearly shows. From these

facts we know that all the classes occurring in any individual, fall most probably on one part (perhaps on one side) of the curve in the racial variation, and that the classes in the individual variation are almost continuous.

1. NINETY PLANTS COLLECTED AUGUST 7

R	2	3	4	5	6	7	8	9	10	Total	D	2	3	4	5	6	7	8	10	11	Total
f	1	4	13	24	27	14	5	1	1	90	f	1	11	18	22	15	16	4	2	1	90

From this point of view, we should take into account the flowers in an equal number of individuals, if we wish to compare two collections in two different conditions, and not an equal number of flowers, because in the latter case we would probably obtain a curve that declines to one side or the other. For example, when in the second collection of rays 1431 heads from 70 plants were taken into account, the mean was 17.777; and when in the third collection 1366 heads from 49 plants were taken, the mean was 17.694; these are due undoubtedly to the smaller number of individuals that were in the first collection.

I have examined also whether the shifting of R and D occurs with the change of the flowering season.

2. NINETY PLANTS COLLECTED AUGUST 11

R	3	4	5	6	7	8	9	Total	D	2	3	4	5	6	7	8	9	Total
f	3	7	25	28	19	5	3	90	f	3	4	20	24	18	11	7	3	90

3. NINETY PLANTS COLLECTED AUGUST 19

R	3	4	5	6	7	8	9	10	Total	D	2	3	4	5	6	7	8	9	10	11	13	Total
f	1	2	9	23	30	14	7	4	90	f	1	2	5	22	29	11	12	5	1	1	1	90

In the second collection only a slight change occurred, while in the third collection the number of different classes in each individual increased to 7 (the most probable one), and accordingly the number of D shifted from 5 to 6. These increases result from the addition of classes of lower value, to which the seasonal change of rays is due.

Correlation between the number of rays and of disk florets

The material of the third collection was used in this calculation. A glance at the correlation surface in the accompanying table shows

that correlation certainly exists in some degree. From the data of this table, I computed by PEARSON'S method the coefficient of correlation between the rays and disk florets of *Aster fastigiatus*. The result of my calculation is $r=0.3219\pm0.0111$. Since the coefficient of correlation always lies between 0 and 1, my result shows a significant correlation, though not in a high degree. Generally the correlation between the rays and disk florets appears not very large (SHULL **12**), so that SHULL found this coefficient only 0.574+−0.353 in *Aster prenanthoides;* while he obtained in the correlation of its rays and bracts 0.8559–0.7986. As to the change of the coefficient of correlation in the flowering season, which WELDON (**11**) and SHULL (**12**) discovered in another species, a future investigation of *Aster fastigiatus* is necessary.

CORRELATION SURFACE

Rays	Disk florets														Totals	Mean of disk florets
	0	1	2	3	4	5	6	7	8	9	10	11	12	13		
10	1	..	..	..	..	..	..	..	..	..	..	..	..	..	1	0
11	..	..	..	..	..	1	2	..	1	..	..	..	..	..	4	6.250
12	..	..	..	2	6	9	7	5	3	..	..	..	..	..	32	5.500
13	..	..	..	1	4	15	44	21	14	3	1	..	..	..	103	6.342
14	..	..	..	..	3	26	43	62	39	10	..	..	..	..	183	6.754
15	..	..	..	..	5	37	52	119	77	19	7	1	..	..	317	6.997
16	..	..	..	1	4	30	109	153	130	66	14	4	..	..	511	7.225
17	..	..	..	..	1	23	72	185	162	76	19	3	..	..	541	7.484
18	..	1	..	..	2	16	58	120	168	83	21	1	1	..	471	7.645
19	..	..	1	..	..	7	53	72	93	52	20	5	..	..	303	7.677
20	..	..	..	..	..	3	25	55	57	27	7	3	..	..	177	7.633
21	..	..	..	..	..	2	15	37	46	24	8	3	..	..	135	7.822
22	..	..	..	..	..	2	2	18	17	10	5	4	..	1	59	8.153
23	..	..	..	..	..	..	9	10	18	8	..	3	..	..	48	7.711
24	..	..	..	..	..	1	1	11	15	3	7	4	..	1	43	8.419
25	..	..	..	..	..	..	..	4	7	2	3	1	..	..	17	8.412
26	..	..	..	..	..	..	1	..	2	..	4	2	1	..	10	9.600
27	..	..	..	..	..	..	..	..	..	1	1	..	..	..	2	9.500
28	..	..	..	..	..	..	..	..	1	..	..	..	1	..	2	10.000
Totals	1	1	1	4	25	172	493	872	850	384	117	34	3	2	2959	

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Summary

1. In the variation of the number of the rays and disk florets of *Aster fastigiatus* Fisch. and Mey., the curve is always monomodal, and its mode does not belong to the Fibonacci series.

2. The seasonal change in the number of rays is clearly observable in this species.

3. All the classes in individual variation have a tendency to fluctuate on one side or other of the curve of the racial variation, and appear to be almost continuous.

4. The number of rays and that of disk florets change correlatively to each other in a tolerably marked degree.

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